THE BB&Y ZERO ENERGY HOUSE: REPORT ON THE SECOND YEAR OF OPERATION AND PERFORMANCE ANALYSIS

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ABSTRACT

BB&Y LLC has performed a deep energy retrofit on a house in Redwood City, California. The purpose of the project is to go beyond net zero energy and explore how a house can operate with zero greenhouse gas emissions. This paper is a follow-on to a previous paper entitled *THE BB&Y ZERO ENERGY HOUSE: REPORT ON THE FIRST YEAR OF OPERATION*.[1] That paper described the project design and the results from the first year of operation. During the second year of operation various modeling and experimental activities took place to better understand the energy flows with the aim of improving performance. These are covered in this paper.

1. INTRODUCTION

As discussed in the first paper on the BB&Y experimental house, the house required only 139 kWh of grid back-up support during the first full year of operation. At the time it exported 7,096 kWh to the local grid. This enabled the house to be a net carbon sink.[1] The data revealed that the winter heating electrical load, which was supplied by a heat pump, was about the same as the remaining building electrical loads during December, January and February. This doubled the winter electric energy requirements and necessitated a PV array twice the size needed to serve the other house loads in winter. This is despite considerable effort in upgrading the building envelope to reduce heat loss.[1] We had expected a somewhat lower heating demand.

Based on the low back-up needs, the 9.1 kW_{dc} array was properly sized to nearly eliminate grid draw. It is rather large for such a small house, however; covering roughly half the floor footprint, and thereby supplied three times the house's annual requirements. Another way to view this is that the array is three times the size needed to make the house net zero electric demand under a net metering scheme (as opposed to net zero carbon emission). Clearly further reduction in heat load will enable a considerable downsizing of the PV array and improve the cost effectiveness of the approach. We subsequently found that many of the walls were not insulated. This was rectified prior to the second heating season. This paper chronicles the measurements undertaken to isolate the heat loss paths, and discusses measures for further improvement.

2. SECOND YEAR RESULTS

[1] THE BB&Y ZERO ENERGY HOUSE: REPORT ON THE FIRST YEAR OF OPERATION II, June 23, 2020.

To start, Table 4 of the companion paper was augmented to include the second year of operation. This is shown in Table 1 below. The first point of note is that there is not much difference between the two years despite the wall insulation improvements. There are, however, some relevant aspects to point out. First, the house energy consumption was in general less in the second year. This will be explored in more detail in the following sections. Second, the PV production was slightly larger in the second year. This is despite there being little production in the month of September due to an inverter failure. The improvement comes from the fact that November had normal insolation compared to the previous year, which was heavily impacted by the California wildfires. In addition, the system was operated in grid-connected mode the entire year compared to several periods of off-grid operation in the first year as discussed in reference 1. Other than that, the differences are what one might expect from annual resource and climate variability.

Month	House use (kWh)		PV production (kWh)		Grid Import (kWh)		Grid Export (kWh)	
	1st Year	2nd Year	1st Year	2nd Year	1st Year	2nd Year	1st Year	2nd Year
Jun	326	228	1670	1668	0	0	1286	1390
Jul	305	227	1577	1649	0	0	1300	1375
Aug	270	264	1026	1383	0	0	793	1084
Sep	222	215	519	197	0	129	260	105
Oct	217	241	511	1007	0	0	244	706
Nov	386	387	393	585	85	42	75	231
Dec	417	398	518	415	15	69	57	4
Jan	408	431	521	569	34	21	105	86
Feb	433	339	687	894	6	0	148	500
Mar	351	321	1009	918	0	0	606	538
Apr	211	244	1300	1273	0	0	1040	972
May	210	210	1445	1441	0	0	1180	1203
Total	3756	3505	11177	11802	139	132	7094	8194

Table 1: Comparison of the first year of operation, June 2018 through May 2019, and the second year, June 2019 through May 2020. The second-year grid import total is exclusive of September during which the inverter was largely down.

Figure 1 presents a graphic comparison of the PV array performance with modeling predictions using Homer.[2] It would appear that Homer does a pretty good job of estimating array performance. It might be noted, however, that there is little evidence of a backside boost from the bifacial modules. Unfortunately there was no instrumentation to separate backside and frontside contribution to array output, so no definitive statement can be made in this regard.

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[2] Homer is a widely used microgrid simulation program, https://www.homerenergy.com. For this calculation we assumed a 9.12 kW_{dc} array with horizontal orientation, 0.3%/C temperature coefficient of efficiency, a 7.7 kW inverter with 95% efficiency, and an 85% system derating factor. These correspond to the as-built design.

These results do point out the necessity to allow for some annual variability in weather and solar resource when designing off-grid systems.

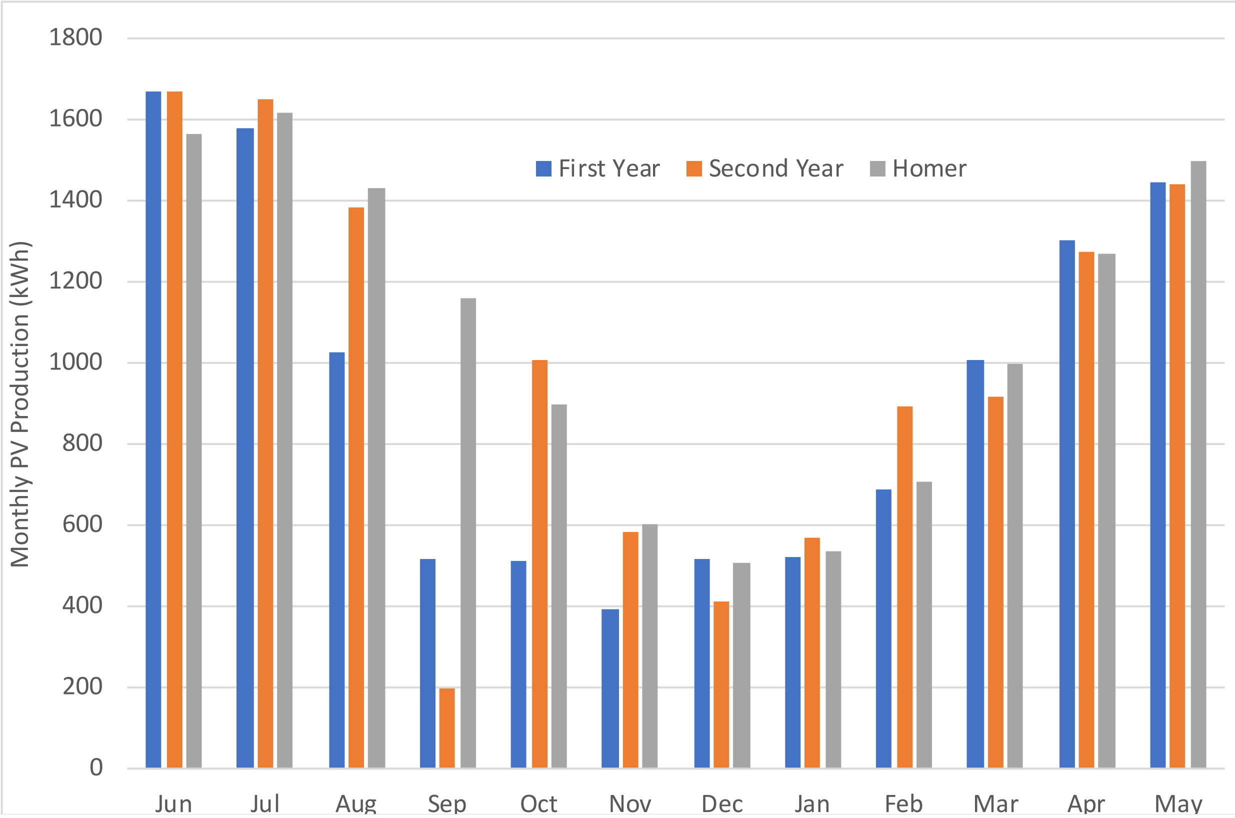


Figure 1: Monthly PV Production comparing the first year, second year, and predictions from Homer. During August, September and October of the first year, the system was operated partially in off-grid mode which reduced PV output. During November of the first year the impact of California wildfires is evident. Finally, during September of the second year the inverter was down most of the month. Other than that, the agreement is quite satisfactory.

To get a view of house operation it is instructive to examine the nicely formatted output from the Tesla Powerwall app. Representative examples are shown in Figure 2. Case (a) is June 21st, the summer solstice. This is pretty much the best case. The battery is charged by 10:00 AM, and thereafter the excess PV is exported. The spikes in house consumption in the afternoon are from the HVAC unit in cooling mode. Case (b) is January 11th, i.e., near the winter solstice. This was a sunny January day, however, and even here the battery become fully charged around noon. The house use spikes are the HVAC unit in heating mode. Moving to case (c) things are different. This was an overcast day and a worst-case situation. The battery never reached full charge the prior day. In fact, in the early morning the battery ran out and grid import was needed as seen by the positive grey. Around 11:00 AM the battery became partly charged and there was no further need for grid support. The figures show that the house is capable of operating in mid-winter with no grid support on clear days. It is the cloudy days that are problematic. In effect, the sizing of the PV array and battery simply determine how many cloudy days in a row the house can tolerate. In the existing design the house is capable of operating about two days in winter with no PV output, and more with some minimal output. It is seen in Table 1 that the house consumption is highest in December and January while the PV output is lowest. Therein lies the rub.

(a) (b) (c)

Figure 2: Representative output from the Tesla Powerwall app. Yellow represents PV output, blue the house use, green the battery power with positive being charging and negative discharging, and grey is the grid power, with positive being consumption and negative export.

a) June 21st, 2020. PV output, 51.4 kWh; house use, 9.6 kWh; grid export, 39.5 kWh, grid import, 0 kWh.
b) January 11th, 2020. PV output, 22.6 kWh; house use, 11.9 kWh; grid export, 9.5 kWh, grid import, 0 kWh.
c) January 20th, 2020. PV output, 10.6 kWh; house use, 16.3 kWh, grid export, 0 kWh; grid import, 2.5 kWh.

BEOpt MODELING

The building modifications described in reference 1 had a significant impact on reducing heating requirements, for they roughly halved the annual heating load. For the entire 2018-2019 heating season the heat pump required 839 kWh. As part of the modeling exercise discussed in the next section, we fit the heat pump published COP versus outside temperature and estimated the total delivered heat to be 4,489 kWh_{th}. Thus the average COP was 5.35. Some building modifications discussed below helped decrease heat load to 3,984 kWh_{th} for the 2019-2920 season. Given that the house has 900 square feet of floor space (83.64 m^2) the annual heating requirement is 48 kWh/m^2. This is 3.2 times the requirements of the Passive House Institute[3] even though the climate in Redwood City is mild compared to most locations where passive houses are considered. Given the diverse climates in the US, the Passive House Institute US developed location dependent standards. For Redwood City it is 7.9 kWh/m^2.[4] So the house consumes six times the Redwood City PHIUS standard. These numbers are compared in Table 2.

BB&Y house	Passive House Standard	Passive House US Standard
(kWh/m^2)	(kWh/m^2)	(kWh/m^2)
48	15	7.9

Table 2: Comparison of the BB&Y house annual heating load with the requirements of the Passive House Standard and the Passive House Institute US standard for Redwood City (actually Moffett Field which is nearby).

With great care it is possible to meet the Passive House US standard in new building construction. It is very difficult to do so in a deep energy retrofit of an existing dwelling, however. Fortunately such measures are not really needed in the Redwood City region. Given that the winter heating electrical load is about the same as the remaining loads for the as-built BB&Y house, eliminating the heating altogether would decrease the PV and battery requirements by 50 percent. Decreasing the heating load by two-thirds would decrease the PV and battery requirements by 33 percent. There is actually little need to go all the way to the 6X reduction required to meet the Passive House US standard, as this would decrease the total only another 8 percent. That would seem to be overkill.

BEOpt made remarkably accurate predictions regarding heating requirements, 0.314 kW average measured versus 0.322 kW average modeled for January. We therefore explored what changes could reduce the heating load further. Some results are shown in Table 3. The various steps are as follows. As built is rather self-explanatory except that we had initially assumed that the walls had R13 fiberglass bat insulation because we observed this in several locations that were opened as part of the remodel. We later discovered that there was no wall insulation in many portions of the house, the exception being where previous remodeling was done, such as by the front bay

[3] https://passiv.de/downloads/03_building_criteria_en.pdf.
[4] https://www.phius.org/phius-2015-new-passive-building-standard-summary. We have converted the quaint Imperial units used by PHIUS into SI units.

window. We subsequently learned that few houses built in the Bay Area in the late 1940s had wall insulation. As shown in Table 3, the house could conceivably be brought into compliance with the passive house standard by installing exterior sheathing, putting the HVAC ducts in a conditioned space rather than in the unconditioned attic, and reducing the remaining air leaks to 1 air change per hour at 50 pascals.

Improvement	kW average	% reduction
As built, uninsulated walls	0.322	0
R13 fiberglass in walls	0.252	22
Above + R12 wall sheathing	0.208	35
Above + ducts to conditioned space	0.155	52
Above + reduce leakage to 1ACH50	0.119	63

Table 3: Modeled average January heat pump power following various building improvements. The heat pump power includes auxiliary resistance heating as well as air handling blower. The various steps are described in the text.

In retrospect, attempts to compare measured results with BEOpt simulations proved rather frustrating. BEOpt uses TMY weather data for Moffett Field, CA. For this weather set, January is the coldest month. We found, however, considerable scatter in heating requirements between years and months. Table 4 shows the average heat pump power for the main heating months during the two years of operation. Also shown are various sources for heating degree-days. For example, January of 2019 was unusually warm, whereas February was unusually cold. On the other hand, February of 2020 was rather warm. From Table 1 it is seen that February 2020 required no backup, and 500 kWh was exported to the grid, compared to only 148 kWh the prior February. Meaningful comparison of simulation that uses TMY weather data versus experiment for the purposes of detailed understanding of building heat load is futile.

			2018-2019		2019-2020	
	TMY Degree Days	WRCC Degree Days	Measured Degree Days	Measured Heat pump Power	Measured Degree Days	Measured heat Pump power
	(F-days)	(F-days)	(F-days)	(kW)	(F-days)	(kW)
December	446	490	427	0.268	449	0.239
January	509	500	422	0.273	463	0.282
February	385	367	533	0.350	337	0.195

Table 4: Comparison of heating degree days and average heat pump power for the three critical heating months during the winters of 2018-2019 and 2019-2020. WRCC refers to

the Western Regional Climate Center data for Redwood City, https://wrcc.dri.edu/cgi-bin/cliMONthdd.pl?caredw. Measured degree days are from a local site of the Weather Underground, https://www.wunderground.com/dashboard/pws/KCAREDWO265 using the normal offset of 65 F.

Prior to the 2019-2020 heating season as a first step in reducing heating requirements we insulated the walls of the house with blown-in fiberglass insulation. Of course we wanted to know what impact this had. Looking at Table 4, it is hard to discern. For example, in January the heating load actually went up! But then that January was colder than the previous one. Additionally, the grid back-up requirement remained essentially the same as shown in Table 1. One might attempt to get a handle on this by taking the total heating load for the three most important heating months and scaling this by the total degree days. The result of this is in Table 5. After this correction, the heating load was reduced by 10 percent compared to non-insulated walls. This compares to the 22 percent reduction predicted by BEOpt as shown in Table 3. Rather short of the desired mark. It should be admitted that this procedure is rather crude and contains large uncertainties. For example, the degree days are calculated with a 65 F offset temperature. This is the temperature at which the house is assumed to require no heating when its internal temperature is 68 F. The 3 F difference is to account for internal gains from appliances, occupants, and solar gain, and is generally used within the HVAC industry. An efficient house such as the BB&Y one actually has a lower offset temperature. In our case it is around 63 F as we'll see in the next section. In addition, solar gain is a very significant source of heat, and so one must also allow for variations in that when computing heat pump load.

	kWh	DD	kWh/DD	% Reduction	
2018	638	1382	0.461	0	As Built
2019	519	1249	0.415	10	Install R13 Wall Insulation

Table 5: Total heat pump energy requirement for December, January, and February before and after installation of R13 wall insulation. When dividing by the actual number of degree days the period after insulation shows a 10 percent heat requirement.

3. TOTAL HEAT LOSS MEASUREMENTS

In order to better understand the sources of heat loss it is clear that measurements based on actual weather data are required. Our first step involved measuring the whole house heat loss coefficient using real time operational and weather data. This section covers the methodology for that. As a second step we then measured the heat transfer coefficient per unit area for each portion of the envelope and computed the theoretical total heat loss coefficient. This is covered in the next section. As will be seen, these two approaches agreed quite closely.

The house's heat loss coefficient, called the UA factor, relates the power loss to the difference in inside versus outside temperature

$$P_{loss} = UA(T_{set} - T_{amb}).$$

We are using the terminology of Randolph and Masters.[5] Here P_{loss} is the heat loss power, T_{set} is the indoor set temperature (68 F in what follows) and T_{amb} is the outdoor temperature, and UA is the total heat transfer coefficient. If the building envelope is comprised of a number of elements of area A_i each with an areal heat transfer coefficient U_i, then $UA = \sum_i U_i A_i$. There are also losses due to foundation perimeter and air infiltration that are bundled into UA, as they are also proportional to the temperature difference.

Since the interior temperature is constant, the energy stored in internal thermal mass is constant. Thus a first law analysis using the house's external surface as the control volume can be written

$$P_{loss} = P_{heat\ pump} + P_{internal}$$

where $P_{heat\ pump}$ is the power deposited in the house as heat from the heat pump and $P_{internal}$ is all the other sources of internal heat, which we will categorize as appliance and plug loads, internal occupant heat, and solar heat gain.

Integrating the above equation over one day gives the total heat loss per day,

$$\int_0^{t_{day}} P_{loss}\, dt = t_{day} P_{loss,ave} = UA\left(\int_0^{t_{day}} T_{set} dt - \int_0^{t_{day}} T_{amb} dt\right) = UA\big(t_{day} T_{set} - t_{day} T_{ave}\big)$$

where the average temperature for the day is

$$\mathrm{T_{ave}} = \frac{1}{\mathrm{t_{day}}} \int_0^{\mathrm{t_{day}}} \mathrm{T_{amb}\, dt}$$

and similarly

$$P_{loss,ave} = \frac{1}{t_{day}} \int_0^{t_{day}} P_{loss}\, dt.$$

All this is to show that the heat loss equation we started with can be written in terms of daily averages

$$P_{loss,ave} = UA(T_{set} - T_{ave}).$$

On a daily basis, the above gives $t_{day} P_{loss,ave} = Q_{loss,day} = UA t_{day}(T_{set} - T_{ave})$. This is the form we used because we had only daily totals from the heat pump. The time units used are

[5] John Randolph and Gilbert Masters, *Energy for Sustainability*, 2008, Island Press, pp. 245.

hours, so that $Q_{loss,day}$ is expressed in kWh/day and t_{day} = 24 hours. Note: based on our experience, we recommend that future work along this line should use hourly data for power and temperature, rather than the daily average we had access to.

In performing the above integration we have ignored the fact that the building exterior will change temperature throughout the day, and some energy will be stored and released there (as opposed to the interior which has a constant temperature). It is easily shown that if the daily temperature profile is periodic then the heat released from the exterior wall integrates to zero over a daily period. If the temperature is not periodic, then our approach neglects the energy that could be stored in the wall and affect the interior the following days. The effect of this is quite small unless the exterior is purposely constructed with high thermal mass. In any case, by averaging over many days the impact of wall storage should average out. Another caveat is that this method is not applicable if the interior temperature ever rises above the set point, 68 F in our case. This restricts it to the main heating months.

The house had a monitor of daily heat pump electrical input power. What we need, however, is heat pump output power into the house. After some juggling, we found that the best approach was to model the variation in heat pump coefficient of performance (COP) using the manufacturers data on COP versus exterior and interior temperature. The equation for the linear fit over the range of interest is $0.1T_{amb} + 0.1$, where T_{amb} is expressed in F. The approximation for the COP was used in the modeling. Occasionally the heat pump would rely on an auxiliary resistance heater. When this happened the auxiliary power was added to the above.

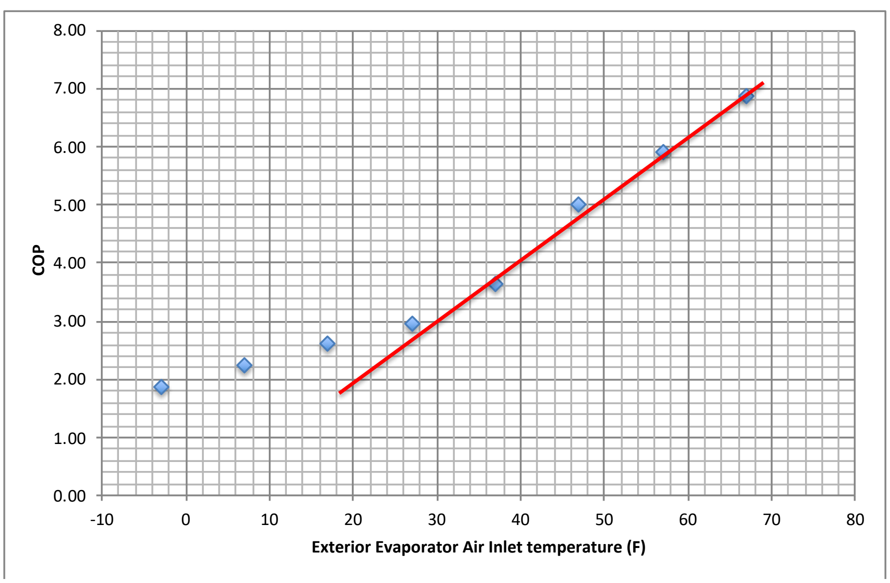


Figure 3: Coefficient of performance (COP) of a Carrier Infinity 25VNA variable speed heat pump for a 68 F condenser air inlet temperature (blue diamonds). The red line is the fit used in this work.

Using the above we plotted the daily heat load versus average daily temperature of all days of December, January, and February in the 2018-2019 heating season in Figure 4. We had to pick an outdoor temperature for which to evaluate the COP. Since the heat pump operates mainly at night, we found that the best results were obtained using the average of the daily minimum and daily average temperature. As expected, the heating load goes up as the temperature goes down; however, the result has a lot of scatter compared to the linear fit expected from the above equations. It is interesting to note that the data appears to converge to zero heat at 63 F. This is in contrast to the standard HVAC practice of assuming 65 F in the degree-day tabulation.

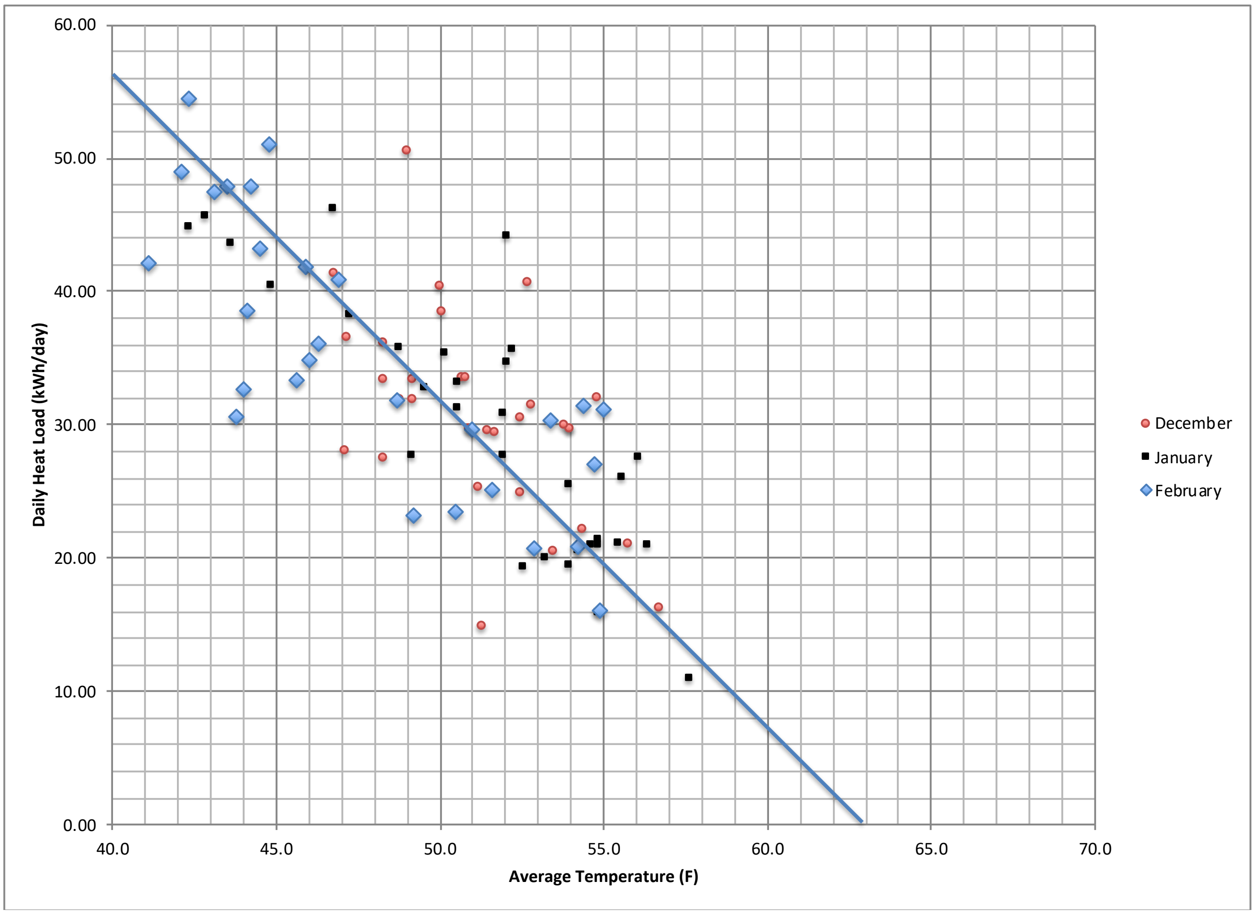


Figure 4: Daily heat load versus average temperature for the 2018-2019 heating season.

Clearly, it is necessary to include other sources of heat besides the heat pump. In looking at some of the outlier points in Figure 4, one finds that they have unusually low or high solar insulation, for example. We considered three such sources: solar gain, internal electrical loads such as stove and plug loads, etc., and the heat from occupants. Since we didn't have any measure of solar gain, we used the PV array output as a proxy for insolation. From the array output in kWh, we computed the total energy falling on the house planform using known array efficiency and area. We then assumed a certain fraction of this made its way into the house. The house was not designed to have much solar gain, having rather small windows, but there is some. We then added this, as well as internal electrical use (excluding heat pump which is outside) and an assume 1.3 kW/day of occupant heat. This is 100 watts ever 24 hours with a 50% occupancy. We then varied the solar gain fraction to find the minimum R^2 value for a linear fit. The gain fraction giving the best fit was 0.05, i.e., 5% of the sun's energy falling on the house makes its way inside. This seems reasonable. After doing this the results of Figure 5 obtain. Clearly the fit is much improved. Plus, the power extrapolated to zero occurs at the set point of 68 F, as it should. The R^2 value for this plot is 0.84. In order to get even improved correlation it is likely necessary to use hourly, rather than daily, data.

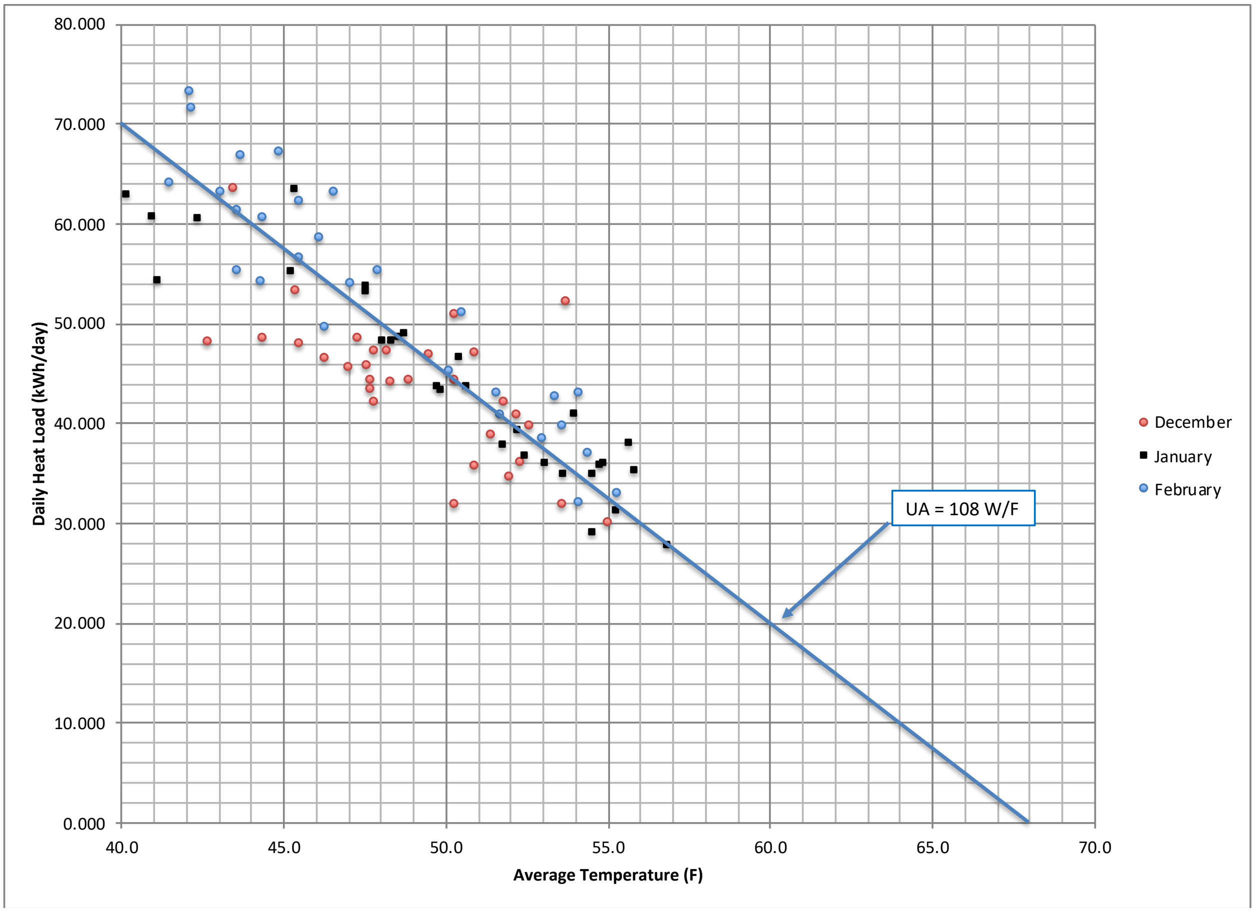


Figure 5. Daily heat load versus average temperature for the 2018-2019 heating season, including solar gain, internal electrical loads, and occupant heat.

The data show that UA = 108 W/F.[6] In other words, or every degree F decrease in average daily temperature, the building power loss increases 108 W.

[6] With apologies for the mixed SI and Imperial units.

We repeated this procedure for the 1019-2020 heating season and obtained UA = 97 W/F. Interestingly this is a 10% decrease following wall insulation, the same result obtained by correcting the monthly heating by degree days, as shown in Table 5. Clearly a lot of work for not much useful new information, except we do now have the building UA factor to compare with the heat loss measurements in the next section.

Table 6 summarizes these findings, along with displaying UA in other units. In the US, UA is usually expressed in units of BTU/hr-F. This is also often converted to the building thermal index by diving by the plan area in square feet and multiplying by 24 hours to get a daily number. These are shown in Table 6. For comparison, reference 5 states that old houses often have a Thermal Index around 15, while super-insulated houses can be down around 4. This jibes with our finding that we halved the losses of the original house, but we are still a factor of two or so above the modern super-insulated standard.

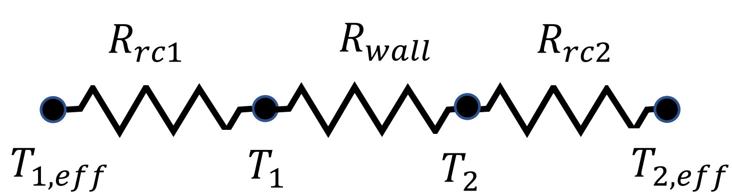
	UA		Thermal Index
	W/F	BTU/h-F	BTU/day-ft^2-F
As Built	108	369	9.8
After Wall Insulation	97	331	8.8

Table 6: Comparison of the UA factor and thermal index in various forms.

4. MEASURING HEAT LOSS MECHANISMS

As discussed above, the heat loss was greater than we suspected. We therefore set upon trying to determine the causes. The following describes the procedure used to measure heat fluxes.

Given a wall or other planar surface in thermal equilibrium on one side with an environment at temperature T_1 and on the other side with an environment at T_2 we can model the heat flow rate between 1 and 2 by the following thermal resistance network.



Here $T_{1,eff}$ is the effective thermal environment temperature on side 1, $T_{2,eff}$ that on side 2, T_1 is the wall surface temperature on side 1, and T_2that on side 2. Correspondingly, R_{rc1} is the combined radiative and convective heat transfer resistance between the wall surface and the surrounding environment, while R_{rc2} is that on side 2. R_{wall} is, of course, the heat transfer resistance of the wall itself.

The heat flux per unit area, q, going from 1 to 2 is then

$$q = \frac{T_{1,eff} - T_{2,eff}}{R_{rc1} + R_{wall} + R_{rc2}} = \frac{T_{1,eff} - T_{2,eff}}{R_{eff}} \ ,$$

where we have defined the effective heat transfer coefficient $R_{eff} = R_{rc1} + R_{wall} + R_{rc2}$. Also we can write

$$q = \frac{T_{1,eff} - T_1}{R_{rc1}} = \frac{T_{1,eff} - T_{2,eff}}{R_{eff}} \tag{1}$$

Solving this equation for R_{eff} gives

$$R_{eff} = R_{rc1} \frac{T_{1,eff} - T_{2,eff}}{T_{1,eff} - T_1} \tag{2}$$

We can thus measure the heat flux and R_{eff} (i.e., the structure's R value) by measuring the indoor, outdoor, and wall surface temperatures using Equations (1) and (2) with an assumed radiative/conductive thermal resistance, R_{rc1}.

The wall temperature is straightforward to measure using an infrared thermometer. Not so the effective interior and exterior temperature, $T_{1,eff}$ and $T_{2,eff}$. While conductive heat transfer responds mostly to the nearby air temperature, the radiative environment can include contributions from many sources. For example, an inside surface receives radiation from the floor, ceiling, walls, doors, and windows all of which are often at different temperatures. $T_{1,eff}$ thus depends on where we are making the R value measurement, as well as the orientation of the surface. When measuring the heat flow through a floor, the floor surface sees the ceiling and walls, but not the floor. Outside it includes radiation from nearby structures, the ground and sky, etc. Often the sky temperature is very low.

From the above, however, it is clear that if the wall resistance, R_{wall}, goes to infinity then $T_1 = T_{1,eff}$ and $T_2 = T_{2,eff}$. To put it another way, $T_{1,eff}$ is the temperature at which a surface that is extracting no net heat is in convective and radiative thermal steady-state with the environment it faces. In Appendix A, we demonstrate that a panel which has an emissive front, insulating interior, and low emissivity back which is placed near the point of interest attains a temperature very near $T_{1,eff}$.

Several 8 ½ by 11-inch panels were constructed using R5 foam board insulation, with aluminum foil on the back as a radiation shield and white paper on the front for the emissive surface. These are shown in Figure 6. The 4-inch pegs on the back are to space the panel a fixed distance from the surface under measurement. This is sufficient to place the panel outside the convective boundary layers of the wall and panel so that the backside air thermal conduction is referenced to an air temperature not affected by the surface under measurement. Figure 7 shows a panel positioned to measure the heat flow near the bottom of a wall.

Figure 6: Panels used for measuring effective environment temperature.

Figure 7. Panel in position to measure lower wall heat flow.

To test whether this method gives reasonable results, we measured the R-factor of a single pane window on a different residence. The measured temperatures are: panel inside, 68.5 F; window inside, 54.5 F; panel outside, 38.5 F. From equations 1 and 2 this gives, using the radiative-conductive R value of 0.68 F ft^2 h/BTU;

Q = (68.5-54.5)/0.68 = 20.59 BTU/h ft^2

R = 0.68*(68.5-38.5)/(68.5-54.5) = 1.46 F ft^2 h/BTU

This is quite reasonable as there was no wind outside, so the outside R_{rc} is 0.68 like the inside, and the glass R value is easily calculated to be 0.034. The total window calculated R value is then

R = 0.68 + 0.034 + 0.68 = 1.39 F ft^2 h/BTU,

meaning that the measured value is 5% greater than the calculated one.

Wall Measurements

A series of measurements on the BB&Y house were performed using this method on days with outside air temperature less than 40 F. These had to be done very early in the morning before the sun began to warm exterior walls. Some representative results are in Table 7. The interior temperature set point was 68 F. The panel and wall surface temperature were measured with a FLIR infrared imaging camera, Model TG165. Example shots are shown in Figure 8. Notice how the interior wall temperature in the left picture is nearly the same as that of the uninsulated wood door. In the right picture taken in the back bedroom, the wall is much cooler than the panel. In fact the wall over the studs is warmer than the wall between studs. These early measurements indicated that the walls were not insulated in many areas.

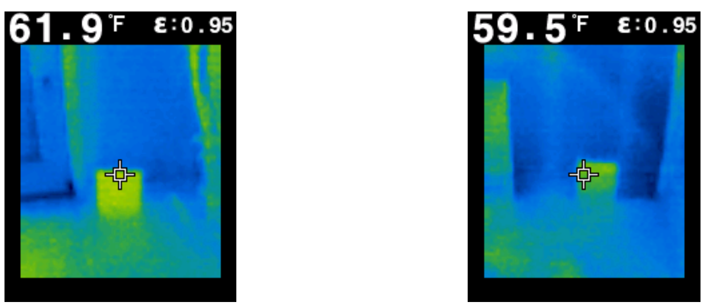


Figure 8: Example infrared images showing the measurement panel in use. The left one is inside to the right of the front door near the floor. The panels in the wood door are visible. The right image is in the back bedroom against an exterior wall. (Note: these images are for illustration only. The actual panel and wall temperatures used in the calculations were measured close to the wall and panel.)

Location	surface T	panel T	otherside T	Q	R
	(F)	(F)	(F)	(BTU/h ft2)	(F ft2 h/BTU)
Wall to right of front door	56.0	62.1	33	8.97	3.24
Back BR, south-east wall	56.5	59.5	33	4.41	6.01
Door panels (meas. from outslide)	44.0	32.6	68	16.76	2.11
Door frame (meas. from outslide)	40.3	32.6	68	11.32	3.13

Table 7: Measured Q and R at several locations.

It is interesting to compare the first result in Table 7 for the front wall, which gives R = 3.24, to the calculations in Reference 5, page 235, which gives R = 3.44 for an uninsulated wall comprised of 2X4s on 16-inch centers. Not bad. The back bedroom wall, however, has an R value intermediate between an uninsulated wall and an insulated one. The reason for this discrepancy is not understood. Based on what was subsequently learned during the perimeter loss measurements discussed below, this is likely due to the fact that the lower portion of this wall was only several feet from a fence, and hence the outside effective temperature higher than the assumed 33 F (the value measured in front of the house). Unfortunately, we discovered this issue too late to make outside effective temperature measurements with a panel opposite to the one inside. A good lesson for the future.

Perimeter Measurements

While performing the above IR measurements, it became clear that there was significant heat leaking out of the foundation perimeter. We made an attempt to quantify this to aid in assessing the overall losses. Figure 9 shows some IR images along the foundation perimeter.

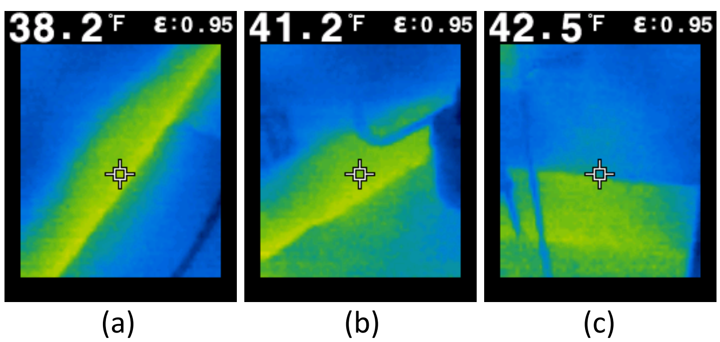


Figure 9: IR images along foundtion perimeter. (a) along south east side of house, (b) along back of house, (c) foundation in garage.

To quantify this we measured the temperature near the foundation at 6-inch intervals as shown in Figure 10. Of note is the fact that the panel temperature depends on orientation. The panel on the ground is looking at the sky and has a lower temperature than the vertical panel by the wall. This panel sees a nearby fence. Also, in retrospect, we should have moved the panel up the wall as we took temperatures higher up because the top reading at 18-inches is cooler than the panel (which is cannot be as heat is exiting the structure). So our measurement clearly underestimated the loss. Also, for the measurements along the ground we didn't go far enough from the foundation because the temperature at 18-inches was still higher than the panel, i.e., heat is still coming out beyond 18-inches. This also underestimates the loss. In any case, we then divided the regions into 6, 6-inch by 1-foot long, strips (as shown by the dashed rectangle in Figure 10) and used the average temperatures on each side as the one to use in Equation 1. In summing these, recognizing that each strip is 0.5 sq. ft., this we obtained 13.6 BTU/h-ft. Given that the difference in indoor and outdoor temperature is 34 F, we get the perimeter F-factor of 0.4 BTU/h-ft-F.

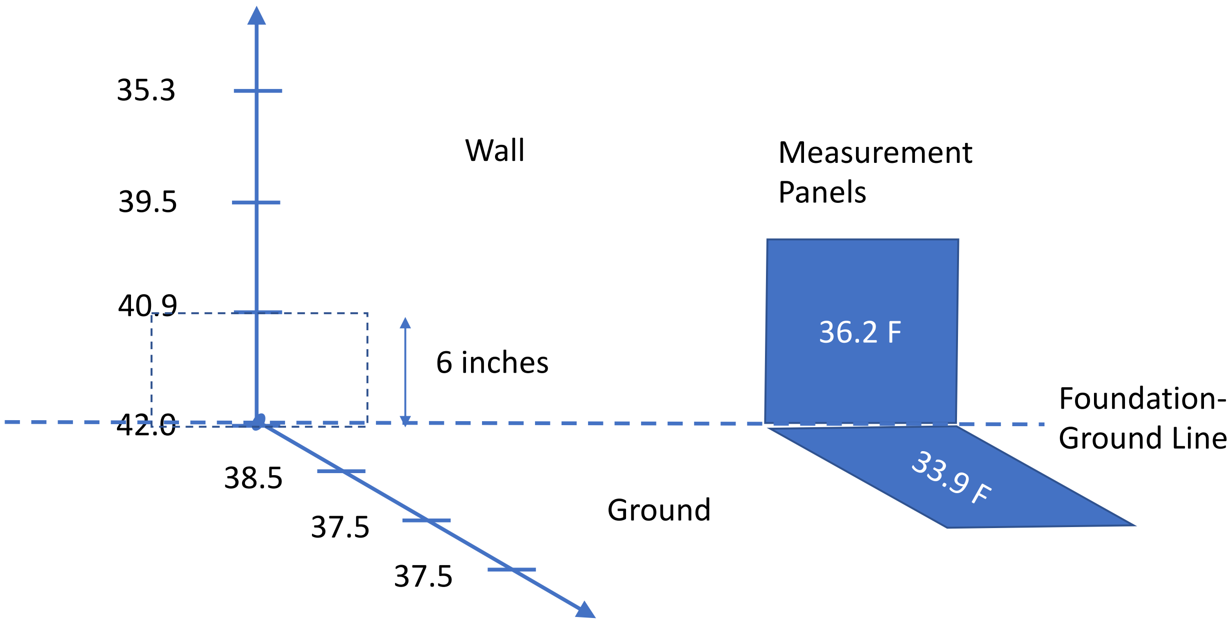


Figure 10: Geometry of foundation temperature measurements. The temperatures, measured every 6-inches from the foundation-ground line, are in F.

This result compares with a recommended F-factor of 0.9 BTU/h-ft-F for an uninsulated crawl space and 0.55 BTU/h-ft-F for a foundation insulated with R-6 on the outside to a depth of 18-inches given in reference 5, page 238. In our case, there is R-18 sprayed insulation on the inside of the foundation wall, but it doesn't go below grade. Heat is clearly conducting down through the earth and heating the foundation footing. So the measured 0.4 BTU/h-ft-F doesn't seem unreasonable, remembering that it is an underestimate. From a literature search, it appears that the conditioned crawlspace with insulation on the inside of the above grade foundation that we have is closest to a slab on grade regarding perimeter losses, and all the studies give results near F-factor is 0.9. Therefore we will use this in modeling and assume that our measurements are about a factor of two low.

The house foundation that is against the garage is somewhat different because it is radiating into the unconditioned and uninsulated garage. As a result, the temperature is more uniform going from ground up to its top 18 inches above the garage floor. In fact, it varies only one degree, going from 48.5 F at the bottom to 47.5 F at the top. The garage effective temperature, as measured by a panel, was 42.9 F. The same procedure as above gives an F-factor of 0.33 BTU/h-ft-F.

5. SIMPLE MODEL OF HOUSE UA FACTOR

We now have the elements to compute the overall UA factor. A simple spreadsheet model was developed to add the various contributions. Figure 11 shows the input and output of this model for the as-built case.

	Areas	R	UA	UA	%	notes
	ft2	F/(BTU/h/ft2)	BTU/h/F	W/F		
Windows						
Living room	72.09	4.39	16.42	4.81	4.46	Dual pane, low E, argon filled, curtain
Front BR bay	29.24	2.50	11.70	3.43	3.18	Est. R, dual pane
Bathroom	10.50	3.23	3.26	0.95	0.88	Marvin spec.
Back BR	17.50	3.23	5.43	1.59	1.47	Marvin spec.
Kitchen	3.83	3.23	1.19	0.35	0.32	Marvin spec.
Patio Door/Window	34.52	4.69	7.36	2.16	2.00	Marvin spec. + R=1.36 curtain
Front Door	17.78	2.60	6.84	2.00	1.86	Est. R
Kitchen-Garage Door	16.67	5.20	3.21	0.94	0.87	R doubled due to looking into garage
Wall to garage	211.60	12.66	16.71	4.90	4.54	Reference 5, p. 235
Front BR BR wall	93.96	12.66	7.42	2.17	2.01	Reference 5, p. 235
Walls (less above)	518.62	6.90	75.16	22.02	20.40	See discussion
Ceiling	909.70	30.00	30.32	8.88	8.23	Cellusose
Ducts	141.00	8.00	17.63	5.16	4.78	90', 6" dia, R8
Subtotal			202.63	59.37	55.01	
Perimeter			105.01	30.77	28.51	
Infiltration			49.68	14.56	13.49	
HRV			11.04	3.23	3.00	
Total			368.36	107.93	100.00	
Measured UA/F			368.60	108.00		
Perimeter calc.						
Perimeter in garage	26.45	ft				
Perimeter remaining	101.98	ft				
Garage	0.50	BTU/h/ft/F				
Remaining	0.90	BTU/h/ft/F				
UA	105.01	BTU/h/F				
Infiltration calc.				HRV	100.00	CFM
CFM50	900.00	cfm		HRV eff	90.00	%
ACH50	5.94	1/h		HRV loss	11.04	BTU/h/F
Natural conv. Derate	20.00					
ACH actual	0.30	1/h				
CFH actual	2700.00	ft3/h				
Cp*rho	0.018	BTU/ft3/F				
UA	49.68	BTU/h/F				

Figure 11: Simple spread sheet model of building losses based on the methodology in Reference 5. Yellow cells are input data. Red is the overall UA.

While comparing this model to the measured UA, we ran into some difficulty in that the uninsulated wall R-factor needed to get agreement is 6.0, as seen in Figure 11. This is nearly double the 3.44 as calculated in Reference 5. The reason for this is not certain. A contributing factor could be that the effective room temperature is lower than the thermostat set temperature when the walls are lossy. The thermostat is in the central portion of the house away from the back bedroom, which had the largest contribution of uninsulated walls. Note that the effective temperature for the back bedroom in Table 7 is only 59.5 F, which is lower than measured in any other room. It is interesting that the R-factor of 6.0 needed to get agreement is very close to the measured value of 6.01 for the back bedroom (Table 7). In any case, with this R-value, the computed UA is 108 W/F, the same as the measured value.

Another finding was that we had to double the perimeter F-factor from the measured value of 0.4 BTU/h-ft-F to 0.8 BTU/h-ft-F. As mentioned above, this measurement was surely an underestimate. It did show that perimeter loss is a significant factor, however. In doing a literature search on the matter, we found that there is wide agreement that the number for an uninsulated slab foundation is near 0.9 BTU/h-ft-F, as used in Reference 5. Even though we do have insulation on the interior vertical portion of the crawlspace wall, there is no below ground or lateral insulation. It would seem that the crawlspace floor, therefore, is most similar to an uninsulated slab as far as heat transport goes. In addition, most slab calculations assume no conducting wall above grade, which we have in the concrete crawl space foundation. We saw considerable heat coming from this wall even though it was insulated on the inside. Lacking further data, it is not unreasonable to use 0.9 BTU/h-ft-F.

Under these assumptions it is seen that the uninsulated walls account for 23% of the calculated heat loss. As discussed in the section on total heat loss measurements, after insulating the uninsulated walls, we measured UA to be 97 W/F. Putting in R=12.66 for the insulated stud walls instead of 6.0 in the spreadsheet gives UA = 98 W/F, again almost equal to the measured value. This agrees with the 10% measured in Section The originally uninsulated wall heat loss drops from 23% to 12% of the total heat loss.

6. DISCUSSION

So it seems that we have an explanation why BEOpt predicted twice the improvement expected from insulating the uninsulated walls. And we have learned some interesting things in the process. First, the actual non-wall losses are likely larger than BEOpt predicted. Second, the loss through uninsulated walls is less than BEOpt predicted. These combined to make insulation appear to have more of an impact than in actuality.

Table 8 show the computed relative contributions from various loss paths. While there remains considerable uncertainty in the exact heat loss contributions, the following conclusions seem relatively trustworthy.

Loss path	%
Perimeter	28.4
Walls	19.5
Infiltration	14.9
Windows	13.6
Ceiling	9.1
Doors	6.1
Ducts	5.3
HRV	3.3
Total	100.0

Table 8: Heat loss paths by percentage contribution during the second year of operation.

As shown in Table 8, by far the largest component of heat loss, perhaps surprisingly, is the perimeter loss, which of course doesn't respond to wall insulation. One mitigation strategy could be to install horizontal perimeter insulation on the ground. Unfortunately the literature suggests that various types of foundation insulation have marginal impact.[7] For example, 4 feet of vertical below grade perimeter insulation decreases the computed F-factor from 0.86 to 0.60 BTU/h-ft-F in the calculation of Reference 7. While such a reduction is significant, the perimeter loss would still be very high.

It would seem that BEOpt underestimated perimeter loss. Perimeter loss is not spelled out specifically in BEOpt output, so we tried to get an estimate of its impact by putting in R-40 floor insulation in and ventilating the crawlspace. Comparing the result with and without such, and attributing the difference to perimeter loss, gives the loss at 27.3 BTU/h-F, or F=0.23 BTU/h-ft-F. This is only 29% of the value in Figure 11.

The second major difference is that BEOpt seems to overestimate the heat loss from uninsulated walls located in rooms that are remote from the thermostat. That is because the room runs cooler than the area near the thermostat. This is supported by the tenant who said the back bedroom felt much warmer following insulation. This is only one reason why calculated wall losses can be off. Others include radiation for external structures and vegetation and temperature variations with height. These issues are apparently well known in the building modeling community.[8] For example, Reference 8 quotes "Multiple studies confirm that analysis methods tend to overpredict

[7] D. Baylon and M. Kennedy, "Calculating the Impact of Ground Contact on Residential Heat Loss," https://web.ornl.gov/sci/buildings/conf-archive/2007%20B10%20papers/092_Baylon.pdf.
[8] B. Polly, N. Kruis, and D. Roberts, "Assessing and Improving the Accuracy of Energy Analysis for Residential Buildings," US DOE, July 2011, https://www.nrel.gov/docs/fy11osti/50865.pdf.

energy use and savings in poorly insulated, leaky homes with older mechanical systems, that is, homes most needing energy retrofits."

Some Lesson Learned

In dealing with the issue of predicting heating load, we have found that there are many sources of error. The problem is quite difficult if one wants more than a ball park estimate. Items which are not well handled by the standard methods include:

1. Radiation balance with exterior objects and the night sky.
2. Impacts of wind speed on exterior convective heat transfer coefficient. Generally the number appropriate for 20 miles per hour wind speed is used as a sort of worst case; however, this may not be appropriate for average values.
3. There is a tendency to use outside air temperature in calculating heat load, as this is generally what local weather stations report. Since radiative losses are greater than convective, this is not particularly confidence building. This problem, as well as 1 and 2 above could be mitigated by installing several convective-radiative panels around the exterior to monitor $T_{2,eff}$ at several locations. (Note: this is actually easier to monitor than air temperature which requires radiative shields around the sensor.)
4. Thermostat set temperature is not a reliable indicator of interior effective temperature, $T_{1,eff}$, especially in a lossy house.
5. We observed a large difference between the top and bottom wall temperatures, both internally and externally. Interestingly, while the interior wall temperature was generally 4 F warmer at the top than at the bottom, the exterior temperature was often colder at the top. The former is what one would expect because the heating outlet ducts are in the ceiling, and the ceiling is well insulated. The outside difference was a bit of a surprise because it is opposite to what would occur from natural convection. (Heating of the exterior wall would cause the boundary layer air to rise and become warmer as it rises.) The difference could be due to the fact that the exterior wall sees more sky at the top. Further measurements are needed to better understand what is happening.

During our heat loss measurements used to model UA, we encountered many uncertainties. For example, we found the data from nearby Weather Underground stations often differed by several degrees. This is another reason to install a dedicated weather monitoring system. Also, monitoring wind speed would be helpful in deciding how much forced convection to expect. Plane of building insolation monitors would be helpful too. We also had some concern about using the heat pump manufacturer specs to model the actual heat pump output. A simple way to measure heat output would be useful. Also, it would be more satisfying to use hourly heat pump energy and outside temperature when comparing models and experiment.

Both the perimeter and infiltration loss are major contributors to heat load. Unfortunately both of these have considerable uncertainty. Perimeter loss is particularly hard to pin down due to the three-dimensional nature of the heat flow, variations in soil thermal properties, and large time delays for regions below the surface. There is near seasonal storage as one gets more than 6 feet down. These uncertainties would be eliminated by insulating the floor and not relying on the ground as thermal barrier. Finally, infiltration loss depends on many factors including wind speed, blocking from nearby buildings and landscaping, etc. This can of course be mitigated by simply making the building very tight. Perhaps we should be surprised it all works as well as it does.

From all the above it is clear that there are several modifications which would, with good certainly, considerably reduce losses. These include the changes in Table 3, as well as putting in some horizontal perimeter insulation. Using the literature value for F with 48 inches of perimeter insulation (Reference 7), adding R13 wall sheathing, and reducing the infiltration to 1ACH50 gives the result in Figure 12. This gives a UA of 66 W/F. This is a lot of work for only a 32 percent reduction in heat load. The biggest issue preventing reaching the passive house goal is perimeter loss.

	Areas	R	UA	UA	%	notes
	ft2	F/(BTU/h/ft2)	BTU/h/F	W/F		
Windows						
Living room	72.09	4.39	16.42	4.81	7.34	Est. R, dual pane, low E, argon filled
Front BR bay	29.24	2.50	11.70	3.43	5.23	Est. R, dual pane
Bathroom	10.50	3.23	3.26	0.95	1.45	Marvin spec.
Back BR	17.50	3.23	5.43	1.59	2.42	Marvin spec.
Kitchen	3.83	3.23	1.19	0.35	0.53	Marvin spec.
Patio Door/Window	34.52	4.69	7.36	2.16	3.29	Marvin spec.
Front Door	17.78	4.00	4.45	1.30	1.99	Est. R
Kitchen-Garage Door	16.67	6.00	2.78	0.81	1.24	R doubled due to looking into garage
Wall to garage	211.60	25.66	8.25	2.42	3.69	Reference 5, p. 235
Front BR BR wall	93.96	25.66	3.66	1.07	1.64	Reference 5, p. 235
Walls (less above)	518.62	25.66	20.21	5.92	9.03	Reference 5, p. 235
Ceiling	909.70	30.00	30.32	8.88	13.55	Cellusose
Ducts	141.00	8.00	17.63	5.16	7.88	90', 6" dia, R8
Subtotal			132.63	38.86	59.28	
Perimeter			71.77	21.03	32.08	
Infiltration			8.28	2.43	3.70	
HRV			11.04	3.23	4.93	
Total			223.72	65.55	100.00	
Measured UA/F						
Perimeter calc.						
Perimeter in garage	26.45	ft				
Perimeter remaining	101.98	ft				
Garage	0.40	BTU/h/ft/F				
Remaining	0.60	BTU/h/ft/F	Reference 5, p. 238			
UA	71.77	BTU/h/F				
Infiltration calc.				HRV	100.00	CFM
CFM50	150.00	cfm		HRV eff	0.90	%
ACH50	0.99	1/h		HRV loss	11.04	BTU/h/F
Natural conv. Derate	20.00					
ACH actual	0.05	1/h				
CFH actual	450.00	ft3/h				
Cp*rho	0.018	BTU/ft3/F				
UA	8.28	BTU/h/F				

Figure 12: Modeled heat lo/ss after extensive additional energy retrofits.

It is worth noting that that if we assign the perimeter loss of 0.9 BTU/h-ft-F to an area-related floor loss, the resulting floor R-factor is 8.3 F-ft^2-h/BTU. (The perimeter is 120 ft and floor area is 900 ft^2 so R = 900/0.9*120.) This is rather minimal. Improving this to 0.6 BTU/h-ft-F increases the floor R-factor to R = 12.5 F-ft^2-h/BTU. This is in line with what could be done with minimal floor insulation. In reference 1 it is pointed out that the decision to with a conditioned crawlspace was driven by the large infiltration air paths between the walls and crawlspace. So the best approach for the future needs further analysis. The same can be said for the attic because the wall-ceiling joint also is also very leaky. Plus the attic contains the HVAC and HRV ducts in an unconditioned space. So would we be better off insulating the roof and making the attic a conditioned space?

In conclusion, this experiment has pointed the way toward large improvements that would significantly reduce the required PV and battery sizes. Whether it remains better to do this as a retrofit on an existing leaky building, or would it be better to simply start with new construction, remains an open question? This is a critical issue that needs needs to be analyzed.

APPENDIX A: HEAT FLOW MEASUREMENT METHODOLOGY

A1. MODELING HEAT FLOW IN A ROOM

Suppose we have N surfaces, each with a known temperature, forming an enclosure (room) and which are exchanging thermal radiation with each other. Then it can be shown that the net heat flux per unit area of surface k, which we denote q_k, is given by (Equation 10.48 on page 810 of *Advanced Heat and Mass Transfer*, A. Faghri, Y. Zhang, and J. Howell, Global Digital Press, 2010.). We have switched sign on q so that positive q represents heat transfer from the enclosure.

$$\sum_{j=1}^{N}\left[\frac{\delta_{kj}}{\epsilon_j} - F_{kj}\frac{1-\epsilon_j}{\epsilon_j}\right]q_j = \sum_{j=1}^{N} F_{kj}\sigma\left(T_j^4 - T_k^4\right) \tag{1}$$

Where ϵ_j is the grey body thermal emissivity of surface j, F_{kj} is the view-factor of surface k looking at surface j, σ is the Stefan-Boltzmann constant, and T_j is the temperature of surface j. If we assume that all the surfaces except k have unity emissivity, this equation simplifies to

$$q_k = \epsilon_k \sum_{j=1}^{N} F_{kj}\sigma\left(T_j^4 - T_k^4\right)$$

The total rate of heat lost by surface k is $\dot{Q}_k = A_k q_k$, where A_k is the area of surface k, is then

$$\dot{Q}_k = \epsilon_k \sum_{j=1}^{N} A_k F_{kj}\sigma\left(T_j^4 - T_k^4\right)$$

Note, the effect of setting all the emissivities to unity except for the surface of interest is to eliminate multiple radiation bounces in the enclosure. Such effects are pretty small in a typical room; however, if they are included then the form of the above equation remains the same, i.e., a linear sum of the radiative emission from the other surfaces. It is just that the factors in the sum are different, and must be found by inverting a matrix. (Ibid, page 810.) This does not change any of the conclusions derived below. In fact, for the measurement panel temperature there is negligible probability that any of the radiation it emits will be reflected back to the panel because it is small compared to the other dimensons.

To simplify the notation, let's assume that the surface we are interested in measuring is surface 1, then we have

$$\dot{Q}_1 = \epsilon_1 \sum_{j=2}^{N} A_1 F_{1j}\sigma\left(T_j^4 - T_1^4\right)$$

Where we have used the fact that $F_{11} = 0$. The surface is planar and doesn't see itself.

Next we assume that surfaces are at roughly the same temperature on an absolute scale sense, and write

$$T_j - T_1 = \Delta_{j1}$$

A first order Taylor series of the above gives

$$\dot{Q}_1 = \epsilon_1 \sum_{j=2}^{N} A_1 F_{1j} \sigma 4\, \Delta_{j1} = \epsilon_1 \sum_{j=2}^{N} A_1 F_{1j} \sigma 4\, T_1^3 \left(T_j - T_1\right)$$

Defining

$$U_{rad} = 4\epsilon_1 \sigma T_1^3$$

gives

$$\frac{\dot{Q}_1}{A_1} = U_{rad} \sum_{j=2}^{N} F_{1j}\left(T_j - T_1\right)$$

We can now add in convective heat transfer. $\dot{Q}_{1,conv}/A_1 = U_{conv}(T_{air} - T_1)$ where $U_{conv} = 1/R_{conv}$ is the convective heat transfer coefficient. The total heat rate is then

$$\frac{\dot{Q}_{tot}}{A_1} = \frac{\dot{Q}_1 + \dot{Q}_{conv}}{A_1} = U_{rad} \sum_{j=2}^{N} F_{1j}\left(T_j - T_1\right) + U_{conv}(T_{air} - T_1)$$
$$= U_{rad} \sum_{j=2}^{N} F_{1j} T_j - U_{rad} T_1 + U_{conv}(T_{air} - T_1)$$

Here we have used the fact that $\sum_{j=2}^{N} F_{1j} = 1$, i.e., all radiation leaving surface 1 has to end up somewhere. The heat flow per unit area is modeled by the simple resistor network coupling the surface to the air and other surfaces. The coupling between surface 1 and surface 2 is by a thermal resistance $1/U_{rad}F_{1j} = R_{rad}/F_{1j}$. If there is no heat extracted, and surface 1 is in steady state with its environment, then we can set $\dot{Q}_{tot} = 0$. Calling the resulting temperature for the surface T_{eff} we have

$$0 = U_{rad} \sum_{j=2}^{N} F_{1j} T_j - U_{rad} T_{eff} + U_{conv}(T_{air} - T_{eff})$$

This can be solved for T_{eff} giving

$$(U_{rad} + U_{conv})T_{eff} = U_{rad} \sum_{j=2}^{N} F_{1j}T_j + U_{conv}T_{air}$$

or

$$T_{eff} = \frac{U_{rad}}{(U_{rad} + U_{conv})} \sum_{j=2}^{N} F_{1j}T_j + \frac{U_{conv}}{(U_{rad} + U_{conv})} T_{air} \tag{2}$$

This is the temperature at which the surface is in thermal balance with the environment it faces. Inserting this into the above equation for heat flow gives

$$\frac{\dot{Q}_1}{A_1} = q_1 = (U_{rad} + U_{conv})\left(T_{eff} - T_1\right) = U_{rc}\left(T_{eff} - T_1\right)$$

Where we have defined the radiative-convective heat transfer resistance as

$$U_{rad} + U_{conv} = U_{rc} = \frac{1}{R_{rc}} = \frac{1}{R_{rad}} + \frac{1}{R_{conv}}$$

This demonstrates the assertion in Equation 1 on page 13.

Some typical magnitudes of these resistances are helpful. From the above equation for U_{rad} we have

$$U_{rad} = 4\epsilon_1 \sigma T_1^3$$

When ϵ_1 = 1 and T_1= 20 C (68 F) this has the value U_{rad} = 5.705 W/m^{2}K = 1.005 BTU/h-ft^2-F. Using the values for R_{rc} recommended in Reference 5, page 225, we can construct the following in Table A1. We have assumed that ϵ_1=0.9, as is typical for painted surfaces. These are the values used thought this report.

	R_{rc}	U_{rc}	U_{rad}	U_{conv}	R_{conv}
Wall	0.68	1.47	0.90	0.57	1.77
Ceiling	0.61	1.64	0.90	0.73	1.36
Floor	0.92	1.09	0.90	0.18	5.48

Table A1: Typical radiative and conductive heat transfer coefficients. U is in units of BTU/h-ft^2-F, and R the reciprocal.

Notice that the radiative coefficients are generally larger than the convective ones. This is particularly true for the floor; however, the case shown is only when the floor is cooler than the air, as is in the case in our house.

A2. APPROXIMATING T_{eff} WITH A RADIATIVE PANEL

It was asserted on page 13 that an insulated panel with a low emissivity back will equilibrate to a temperature close to T_{eff}. We are now in a position to see how close. If we place a panel near a wall, or other surface through which heat is flowing we have the situation modeled in Figure A1.

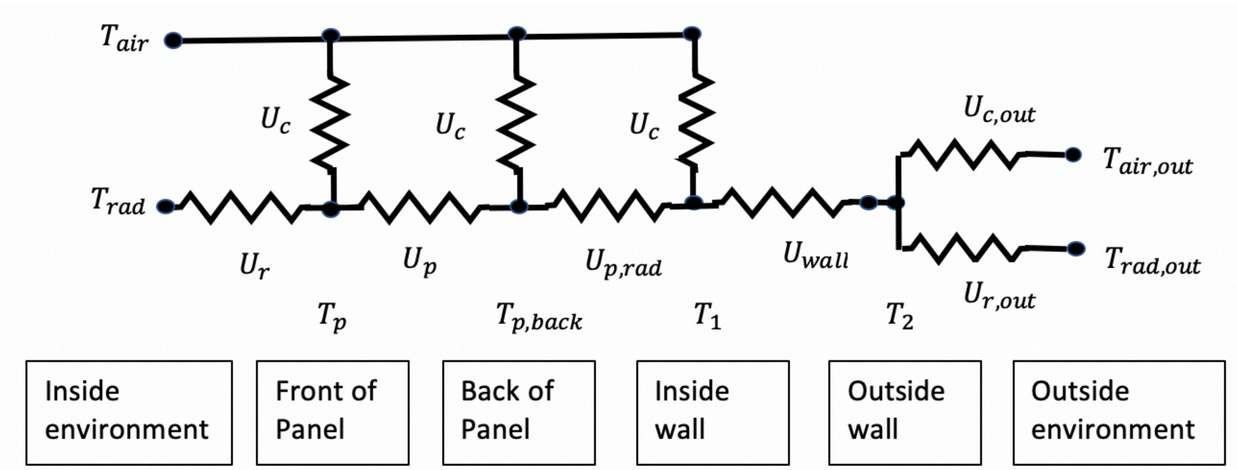


Figure A1: Thermal network with a panel near a wall. The terms are explained in the text.

In Figure A1, we use the following terminology. The heat transfer coefficients are from Table A1. The panel is constructed from R5 foam board with an aluminum foil back having assumed emissivity of 0.1. The U factor between front and back is then 1/5 = 0.2 as shown.

T_{air}	Inside air temperature near wall but outside of convective boundary layer	
T_{rad}	Inside effective radiative temperature. From eq. 2, it is $\sum_{j=2}^{N} F_{1j} T_j$	
T_p	Panel surface temperature facing room	
$T_{p,back}$	Panel surface temperature facing wall	
T_1	Wall temperature near pane	
U_r	Radiative heat transfer coefficient, ε=0.9	0.90
U_c	Convective heat transfer coefficient	0.57
U_p	Conductive heat transfer coefficient from front to back of panel	0.20
$U_{p,rad}$	Radiative heat transfer coefficient for back of panel to wall, ε=0.1	0.10

The panel is placed around 4 inches from the wall, and we assume that the underlying wall temperature is not affected. This is reasonable given that the wall has considerable thermal mass and lateral conductivity, and the panel is only 11 inches on its longest side. We also assume that there is only radiative coupling between the panel and the wall, i.e., no direct air conduction.

Both are in convective connection with the air temperature, which is assumed to be the same between the panel and wall as in front of the panel.

It is obvious from Figure A1 that if the panel is perfectly insulated, $U_p = 0$, then $T_p = T_{eff}$. Likewise, if $U_{p,rad} = 0$, and $T_{air} = T_{rad}$ then $T_p = T_{eff}$. So the panel is affected by radiative coupling to the wall and by convection off its back surface. The question at hand is, how much impact do these factors have?

Summing heat flow into nodes T_p and $T_{p,back}$ gives the following equations.

$$U_r\left(T_{rad} - T_p\right) + U_c\left(T_{air} - T_p\right) + U_p\left(T_{p.back} - T_p\right) = 0$$

$$U_p\left(T_p - T_{p,back}\right) + U_c\left(T_{air} - T_{p,back}\right) + U_{p,rad}\left(T_1 - T_{p,back}\right) = 0$$

Between these equations $T_{p,back}$ can be eliminated giving the value of T_p. The resulting expression is rather cumbersome and won't be displayed. What we really are interested in is how accurately the heat transfer is measured. So we compare the actual heat transfer, $\dot{Q}_{actual}$, to the measured value, $\dot{Q}_{measured}$.

$$\dot{Q}_{actual} = (U_r + U_c)(T_{eff} - T_1)$$

$$\dot{Q}_{measured} = (U_r + U_c)(T_p - T_1)$$

The ratio $\dot{Q}_{measured}/\dot{Q}_{actual}$ is plotted in Figure A2. It is seen that when the air temperature is close to the radiative temperature the results agree well, within 2 percent or so. As the air temperature deviates more the accuracy is less. This is because the radiative panel loses some heat off the backside due to convection. Given the accuracy we are looking for, however, these results seem quite satisfactory. The accuracy could no doubt be improved by using better insulation than R5 in the panel, for example vacuum insultation.

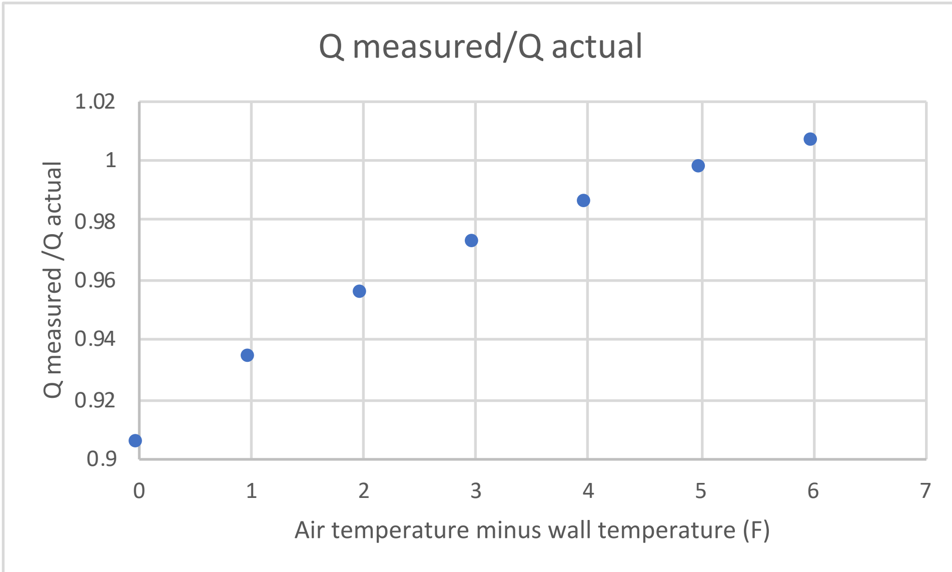


Figure A2: Comparison of actual heat transfer rate to that measured by the panel method. The wall temperature is set at 68 F and the radiative temperature is assumed to be 4 F hotter.